# Fluid flows in a weeping bearing: An approximate calculation

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**Summary**

Unless the separated surfaces of a fluid-lubricated bearing are sliding rapidly past each other they soon touch, high-spot to high-spot, after load is applied unless they are very smooth. At first this contact carries little of the bearing load and causes little friction, but both increase as the bearing load squeezes fluid from the regions surrounding the contacting high spots, which squash down and spread. The period of low friction is prolonged if the bearing material is porous, compressible and fluid-soaked, because it weeps fluid that replaces most of the fluid that is lost. This paper calculates the flows in a weeping bearing, quantifies the effect of compressibility of the high spots, and shows that weeping will continue at least as long as the high spots remain a little taller than the radius of the pores in the bearing material.

## 1. List of symbols

$C_{Ab}$ confined compressive compliance of bulk cartilage normal to its surface measured at constant pore pressure

$C_{Ac}$ confined compressive compliance normal to the cartilage surfaces of the high-spot bridged crack between the rubbing surfaces of the cartilages measured at constant pore pressure

$C_{A(c/b)}$ ratio of the confined compressive compliance of the high-spot bridged crack to that of bulk cartilage

d diameter of the pores in cartilage

$H_{Ab}$ confined compressive stiffness of bulk cartilage normal to its surface measured at constant pore pressure

$H_{Ac}$ confined compressive stiffness normal to the cartilage surfaces of the high-spot bridged crack between the rubbing surfaces of the cartilages measured at constant pore pressure

$H_{A(c/b)}$ ratio of the confined compressive stiffness of the high-spot bridged crack to that of bulk cartilage

$k_b$ permeability of bulk cartilage tangential to its surface

$k_c$ tangential permeability of the high-spot bridged crack between the rubbing surfaces

$k_{(c/b)}$ ratio of the tangential permeability of the high-spot bridged crack between the rubbing surfaces to that of bulk cartilage

π 3.14159 etc.

R radius of each cartilage

S Skeleton stress (total stress minus pore pressure) in the cartilage normal to its surface

T thickness of each cartilage

t height of the asperities on the surface of each cartilage

$V_n$ rate of decrease of the spacing between the bones

$V_b$ radial flow velocity (I. e., volume/[area•time]) of fluid through bulk cartilage at the bearing rim

$V_c$ radial flow velocity (I. e., volume/[area•time]) of fluid through the high-spot bridged crack between the rubbing surfaces at the bearing rim

**2. Introduction**

Consider a fluid-lubricated bearing whose surfaces are separated and not moving rapidly past each other. If the surfaces are pushed toward each other the bearing load is at first carried by pressure in the fluid in the crack between them. Because the fluid escapes radially from the loaded region this pure "squeeze-film" lubrication lasts only until the bearing surfaces touch, typically soon because most bearing surfaces are not smooth. The contact is high-spot to high-spot, leaving passages between the high spots through which fluid can still escape. The contacting high spots carry some of the load, the portion increasing as the fluid surrounding them escapes along the crack. The increasing load makes the high spots squash down and spread. This is "mixed lubrication."

The surface of articular cartilage is not smooth (Gardner, 1972; McCutchen, 1980), yet the low friction lasts many minutes (McCutchen, 1962). Either the fluid in the mixed lubrication is somehow prevented from escaping, for which there is no apparent mechanism, or it is being replaced by new fluid.

Cartilage is porous, deformable and soaked with fluid, which it weeps if squeezed (Lewis and McCutchen, 1959). In a loaded bearing this fluid is at high pressure (McCutchen, 1959). It weeps out through the rubbing surfaces of the cartilages, replaces most of the fluid that escapes along the crack, and carries most of the bearing load for a long time (McCutchen, 1959; McCutchen, 1972).

Maroudas (1967), Walker *et al.*. (1968), Dowson, *et al.* (1970) and Hlavacek (1993) argued that instead of flowing out through the cartilage surfaces fluid would be pushed back into the cartilages by the bearing load. But high-spot to high-spot contact raises the pressure of the fluid within the cartilages relative to that in the crack (Lewis and McCutchen, 1960; McCutchen, 1972).

In McCutchen (1973) I wrote that for "replenishing of the partial squeeze film the resistance of the crack between the cartilages to flow parallel to the cartilage surfaces must be less than that of a slice of bulk cartilage material of the same thickness" I did not realize that this criterion holds only if the confined compressive moduli of crack and bulk are identical. This article derives a proper criterion.

McCutchen (1975) used analog, consolidation-theory computation of the flows in a weeping bearing to see how properties such as the flow conductance along the crack affect the bearing's behavior under intermittent loading. The effect of crack compressibility was ignored. It will be shown below that for likely cartilage roughness and stiffnesses of crack and bulk the effect of crack compressibility (<u>but not crack thickness and thus fluid conductance</u>) on weeping flow is small unless the bearing has been loaded for a long time.

**3. The calculation**

Consider a bearing made of two discs of articular cartilage of uniform thickness T and radius R, pressed together without lateral motion between parallel, flat bone ends. Real bones do not have flat ends, but flat ends simplify the mathematics without changing the important physical effects. To keep a steady grip on reality I substitute at each stage in the mathematics a specific bearing with T = 1 mm, R = 10 mm, and with other specifications introduced as the derivation progresses. General and specific forms of the equations are labeled "g" and "s" respectively.

The cartilage is assumed to be homogeneous. The measured permeability (i.e., 1/resistivity) of bovine articular cartilage normal to its surface is such that if half the volume of cartilage were cylindrical pores, all of the same diameter and all oriented normal to the surface, each pore would be about d = 60 Å = 6 nm = .000006 mm in diameter (McCutchen, 1962). (I am not proposing a structure for cartilage: I am merely estimating the pore size.) A less reliable measurement on the same cartilage sample of permeability parallel to the surface gave a slightly lower value, but I estimated that corrections might double the result. I will assume that the permeability, and thus the pore diameter, for tangential flow are the same as for normal flow. Their precise values are not important to my argument.

Superposed on the thickness T of each cartilage are asperities on its rubbing surface of height t (for tall), assumed to be 600 Å = 60 nm = .00006 mm high. Measured roughnesses are of the order of 10,000 Å = 1,000 nm = .001 mm high (Gardner, 1972; McCutchen, 1980) but load on the asperities can be expected to compress them. Pressed together the two cartilages form a bearing 2T+2t or 2.00012 mm thick. Where high spots on opposing cartilages touch there is zero clearance. Elsewhere are spaces up to 2t or 1200Å = 120 nm = .00012 mm tall that allow fluid to move parallel to the rubbing surfaces.

To estimate the permeability to flow along the crack between the cartilages I approximate the crack by a porous material 2t or .00012 mm thick, with radially-directed cylindrical pores 2t or .00012 mm in diameter. (Again this is not a proposed structure; its only purpose is to estimate permeability.) Values for $k_c$ and $k_b$, the permeabilities of crack and bulk, do not appear individually in this treatment. Only the value of their ratio $k_c/k_b = k_{(c/b)}$ appears. For geometrically similar sponge structures permeability goes as pore diameter squared, so $k_c/k_b = k_{(c/b)} = (2t/d)^2$ or 400. The flow conductance of the crack is the same as that of a slab of bulk cartilage $2tk_{(c/b)}$ or .00012•400 = .048 mm thick.

~~Slopes of cartilage asperities are gentle (M , 1980), so the multi cylinder model probably overestimates flow resistance. If the passages in the crack where rectangular, uniformly 2t or .00012 mm thick, and in the limit occupied all its area, the permeability kc of the crack would be increased by as much as 16/3 and be more favorable to weeping flow. I use the multi-cylinder value in this calculation.~~

If the asperities touch over an area 1/2 times 1/2 = 1/4 of the total area of the bearing the confined compressive stiffness $H_{Ac}$ of the crack might be one fourth that of the skeleton of bulk cartilage normal to its surface $H_{Ab}$. So $H_{Ac}/H_{Ab} = H_{A(c/b)}$ or 1/4. (These are stiffnesses measured for the whole material, solid and fluid, at constant pore pressure.) In this analysis, rather than use confined stiffness it is more convenient to use confined compliance $C_A = 1/H_A$. Thus the compliance of the crack is four times that of the bulk; $C_{Ac} = 4C_{Ab}$, so $C_{A(c/b)} = 4$. As with the permeabilities, values of $C_{Ac}$ and $C_{Ab}$ will not appear individually.

Observed slopes of cartilage asperities are gentle, peaking at 4 degrees (McCutchen, 1980), so the multi-cylinder model, with its large ratio of cylinder wall to flow cross section probably overestimates flow resistance of the crack, perhaps by a factor of 1.5, At the same time the effective compliance of the now-broader asperities is increased by the compliance of the material beneath them, increasing crack compliance by perhaps a factor 6. I use the multi-cylinder values in this calculation.

Collagen fibers are assumed to prevent the cartilages from deforming parallel to their rubbing surfaces. (McCutchen [1978] describes a mechanism by which the high tensile stiffness of collagen fibers can cause weeping outflow before the cartilages touch.)

Assume that the spacing between the bones decreases at a rate $V_n$, where "n" means normal. $V_n$ is in mm/sec when values are substituted. The volume of the cartilages shrinks at a rate $\pi R^2 V_n$ or $\pi \bullet 100 V_n$ $mm^3$ per second. This must equal the rate of fluid escape at the rim of the bearing.

Let $V_b$ be the velocity of fluid issuing radially from the bulk of the cartilage at the rim of the bearing and $V_c$ be the velocity of fluid issuing radially from the crack, both velocities meaning volume flow per unit cross section per unit time. Because fluid in the crack and in the bulk experience nearly the same radial pressure gradient, velocity $V_c = k_{(c/b)} V_b$ or $400V_b$. The total flux from the bearing is the circumference of the bearing times sum of the products of the thickness of bulk and crack by their respective flow velocities, i. e., $2\pi R[2T+2tk_{(c/b)}]V_b$ or $2\pi \bullet 10[2+.00012 \bullet 400]V_b$ $mm^3$ per second. This equals $\pi R^2 V_n$ or $\pi \bullet 100 V_n$ $mm^3$ per second, the rate of volume loss by the cartilages. So the velocity in the bulk is given by

$$V_b = \pi R^2 V_n/\{2\pi R[2T+2tk_{(c/b)}]\}= RV_n/\{2[2T+2tk_{(c/b)}]\}, \quad (1g$$

$$\text{or } 10V_n/\{2[2+.00012 \bullet 400]\} = 2.4414V_n. \text{ mm/sec.} \quad (1s$$

Fluid flows out of the crack at velocity

$$V_c = k_{(c/b)} V_b = k_{(c/b)} RV_n/\{2[2T+2tk_{(c/b)}]\}, \quad (2g$$

$$\text{or } 400 \bullet 2.4414V_n = 976.6V_n, \text{ mm/sec,} \quad (2s$$

so the total flux of fluid from the crack is the circumference of the bearing times the thickness of the crack times $V_c$, i. e.,

$$2\pi R2tV_c = 2\pi R2tk_{(c/b)} RV_n/\{2[2T+2tk_{(c,b)}]\} = \pi R2V_n \bullet 2tk_{(c/b)}/\{[2T+2tk_{(c/b)}]\}, \quad (3g$$

$$\text{or } \pi 100V_n \bullet .00012 \bullet 400/1[2+.00012 \bullet 400] = 7.363V_n \text{ mm}^3\text{/sec.} \quad (3s$$

Some of this crack flow is provided by thinning of the crack. Because the component of pore fluid velocity in the direction perpendicular to the rubbing surfaces is very low the force the fluid applies in this direction to the cartilage solid is very small, and so, therefore, is the gradient of the skeleton stress S perpendicular to the cartilage surface. (S is the stress in the whole material, solid and fluid, minus the pore pressure) And finally, because the thickness of the cartilage is small, S is nearly the same from bone to bone. So, approximately, an increase $\Delta S$ in skeleton stress would accompany a thinning $2T\Delta SC_{Ab}$ or $2\Delta SC_{Ab}$ of the bulk cartilage and a thinning $2t\Delta SC_{Ac} = 2t\Delta S C_{A(c/b)} C_{Ab}$ or $.00012 \bullet 4\Delta SC_{Ab}$ of the crack.

Thinning of the bearing at a rate $V_n$ causes the crack to thin at a rate given by $V_n$ times the fraction of the thinning of the bearing that occurs in the crack, i. e.

$V_n \bullet 2t\Delta SC_{Ab}C_{A(c/b)}/[2T\Delta SC_{Ab}+2t\Delta SC_{Ab}C_{A(c/b)}] = V_n \bullet 2tC_{A(c/b)}/[2T+2tC_{A(c/b)}]$, (4g

or $V_n \bullet .00012 \bullet 4/[2 \div .00012 \bullet 4] = .00023994V_n$ mm/sec. (4s

The volume of the crack declines at a rate given by the area of the bearing, $\pi R^2$ or $\pi \bullet 100\ mm^2$ times the rate of thinning of the crack from Equation 4, i. e.

$\pi R^2 V_n \bullet 2tC_{A(c/b)}/[2T+2tC_{A(c/b)}]$, (5g

or $\pi \bullet 100 \bullet .00023994V_n = .07538V_n\ mm^3$/sec. (5s

The rate of loss in crack volume divided by the crack flow is Equation 5 divided by Equation 3, or

$$= \frac{\pi R^2 V_n \bullet 2tC_{A(c/b)}/[2T+2tC_{A(c/b)}]}{\pi R^2 Vn \bullet 2tk_{(c/b)}/[2T+2tk_{(c/b)}]}$$

$$= \frac{C_{A(c/b)}/[2T+2tC_{A(c/b)}]}{k_{(c/b)}/[2T+2tk_{(c/b)}]} \quad (6g$$

$$\text{or} \quad \frac{4/[2+.00012 \bullet 4]}{400/[2+.000l2 \bullet 400]}$$

$$= \frac{\cancel{1.9995}}{\cancel{195.31}} = \frac{.07538}{7.363} = .010238 \quad (6s$$

For the R = 10 mm, T = 1 mm bearing thinning of the crack provides slightly more than 1% of the crack flow and weeping provides almost 99%. Without weeping, at the same bearing load the crack flow would remain unchanged but would be supplied entirely by crack thinning, which would be speeded up by almost 100 times. Seconds of low friction under load for an impervious bearing become more than minutes for a weeping bearing.

~~Had the crack been twice as thick the 1% would be reduced by about a factor 4, and were the asperities gently sloping there would be a further reduction by a factor not exceeding 16/3.~~

Assuming gentle slopes at the cartilage surface would multiply k(c/b) by 1.5. and C(c/b) by 6. increasing the 1% to 4%. Doubling the crack thickness would divide. the 4% by 4 dropping it back to 1%.

This treatment ignores the resistance to weeping flow that occurs in the normally-directed component of flow within the bulk of the cartilages and within the crack, the latter being much the smaller. Because the diameter of the cartilages is much greater than their thickness, the path of this flow is short and broad. Crudely I estimate its resistance to be only .0006 of the resistance to radial flow along the crack for the R = 10 mm, T = 1 mm bearing. (Here again I assume that the permeability normal to the surface of cartilage is the same as that tangential to the surface. Even a substantial difference in permeabilities would leave the above ratio still very small.)

## 4. Discussion

So long as $C_{A(c/b)}/k_{(c/b)} < 1$ fluid flows from bulk to crack. Had the compliance of the crack been $400C_{Ab}$ rather than $4C_{Ab}$, $C_{A(c/b)} = 400 = k_{(c/b)}$, and Equation 6g would have equaled exactly 1. The crack would have thinned at a rate that exactly supplied the crack flow. There would at that moment be no flow through the rubbing surfaces of the cartilages, but the crack would thin proportionally much faster than the bulk with the passage of time. Had the crack compressibility been higher still, $C_{A(c/b)}/k_{(c/b)} > 1$, and fluid would be displaced by crack-thinning faster than it escaped along the crack; ergo some of it would flow into the bulk.

In the unlikely event that $C_{A(c/b)} = 1$ the criterion for zero flow across the cartilage surfaces is $k_{(c/b)} = 1$, as wrongly given without the requirement that $C_{A(c/b)} = 1$ in McCutchen (1973).

If the crack stiffness is zero the crack becomes a squeeze film between permeable solids as described by Maroudas (1967), Walker *et al.* (1968) and Hlavacek (1993). These authors ascribe cartilage lubrication to an ultrafiltered gel of synovial fluid solute left on the rubbing surfaces by the fluid that enters the permeable cartilages from the squeeze film. But cartilage is still slippery when lubricated by water (McCutchen, 1962), though not as as slippery as when lubricated by synovial fluid.

This paper's mathematics describes the situation at a single moment. As time passes with the bearing under load $k_c$, $k_b$, $C_{Ac}$, $C_{Ab}$ and $V_n$, will all fall as crack and bulk get thinner. In the likely case that $C_{Ac} > C_{Ab}$, i.e., $C_{A(c/b)} > 1$, the crack will thin faster proportionally than the bulk, so $k_c$ will drop relatively faster than $k_b$, and the crack flow will fall relative to that in the bulk, which will also fall. But the squashing down and spreading of the high spots will make $C_{Ac}$ fall toward $C_{Ab}$, making $C_{A(c/b)}$ fall eventually to near 1, so the crack thickness would have to fall nearly to the bulk's pore diameter, making $k_{(c/b)}$ fall closer still to 1, to make $C_{A(c/b)}/k_{(c/b)} > 1$ and send fluid from crack to bulk.

This assumes that the crack can be represented by cylindrical tubes. If the crack has gentle slopes its thickness will have to fall even further for the flow through the cartilage surface to reverse.

Following the changes in S, T, t, $k_c$, $k_b$, $C_{Ac}$, $C_{Ab}$ and $V_n$ to predict exactly the decline in weeping with time is a project for the future.

## Conflict of interest statement, etc.

The work has received no outside support from any source. There is no conflict of interest. The material of this manuscript has not been published before nor presented at a conference, ~~nor will it be submitted elsewhere except as an abstract~~. The work does not involve experiments on humans or animals.

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